

به نام او

ریاضی مهندسی

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$$X(x) = a \cos kx + b \sin kx$$

$$a=0, \quad k = \frac{n\pi}{L} \quad n \in \mathbb{N}$$

$$\Rightarrow \frac{T''(t)}{T(t)} = -C^2 K^2 = -\left(\frac{Cn\pi}{L}\right)^2$$

$$\Rightarrow T(t) = \alpha \cos\left(\frac{Cn\pi}{L}t\right) + \beta \sin\left(\frac{Cn\pi}{L}t\right)$$

$$\lambda_n := \frac{Cn\pi}{L} \quad [L_n \text{ ویره،} \text{ لیس}]$$

$$\{\lambda_n \mid n \geq 1\} \quad \text{نکتہ}$$

$$X(x)T(t) = (\alpha_n \cos(\lambda_n t) + \beta_n \sin(\lambda_n t)) \times \sin\left(\frac{n\pi}{L}x\right) = u_n(x,t)$$

$$\sum_{n=1}^{\infty} (\alpha_n \cos(\lambda_n t) + \beta_n \sin(\lambda_n t)) \sin\left(\frac{n\pi}{L}x\right) = u$$

$$u(x,0) = f(x)$$

$$u_+(x,0) = g(x)$$

$$f(x) = u(x, 0) = \sum_{n=1}^{\infty} (\alpha_n + 0) \sin\left(\frac{n\pi}{L}x\right)$$

$$g(x) = u_t(x, 0) = \sum_{n=1}^{\infty} (\beta_n \lambda_n) \sin\left(\frac{n\pi}{L}x\right)$$

$$\Rightarrow \alpha_n = \frac{1}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

$$\beta_n = \frac{1}{L \lambda_n} \int_0^L g(x) \sin \frac{n\pi x}{L} dx = \frac{1}{c n \pi} \int_0^L g(x) \sin \frac{n\pi x}{L} dx$$

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$$\sum_{n=1}^{\infty} (\alpha_n \cos \lambda_n t + \beta_n \sin \lambda_n t) \sin\left(\frac{n\pi}{L}x\right)$$

$$\alpha_n = \frac{1}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

$$\lambda_n = \frac{cn\pi}{L}$$

$$\beta_n = \frac{1}{cn\pi} \int_0^L g(x) \sin \frac{n\pi x}{L} dx$$

$$c \sin \lambda_n t + \sin \frac{n\pi}{L} x = \frac{1}{r} \left(\sin \left(\frac{n\pi}{L} x + \lambda_n t \right) + \sin \left(\frac{n\pi}{L} x - \lambda_n t \right) \right)$$

اُن سُرَجَت اِرْلِيْمِ مُفْرِبَة

$$u(x, t) = \sum_{n=1}^{\infty} \alpha_n \left(\sin \left(\frac{n\pi}{L} x + \lambda_n t \right) + \sin \left(\frac{n\pi}{L} x - \lambda_n t \right) \right)$$

$$= \sum_{n=1}^{\infty} \alpha_n \left[\sin \left(\frac{n\pi}{L} (x + ct) \right) + \sin \left(\frac{n\pi}{L} (x - ct) \right) \right]$$

$$= \frac{1}{r} \left[f^*(x+ct) + f^*(x-ct) \right] \stackrel{f^* \text{ is even}}{\equiv} \frac{1}{r} (f(x+ct) + f(x-ct))$$

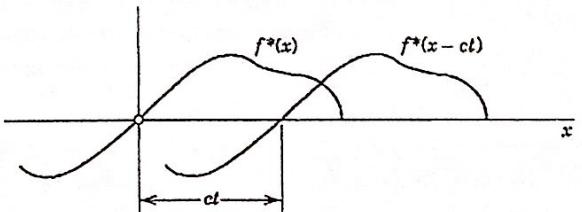


Fig. 287. Interpretation of (17)

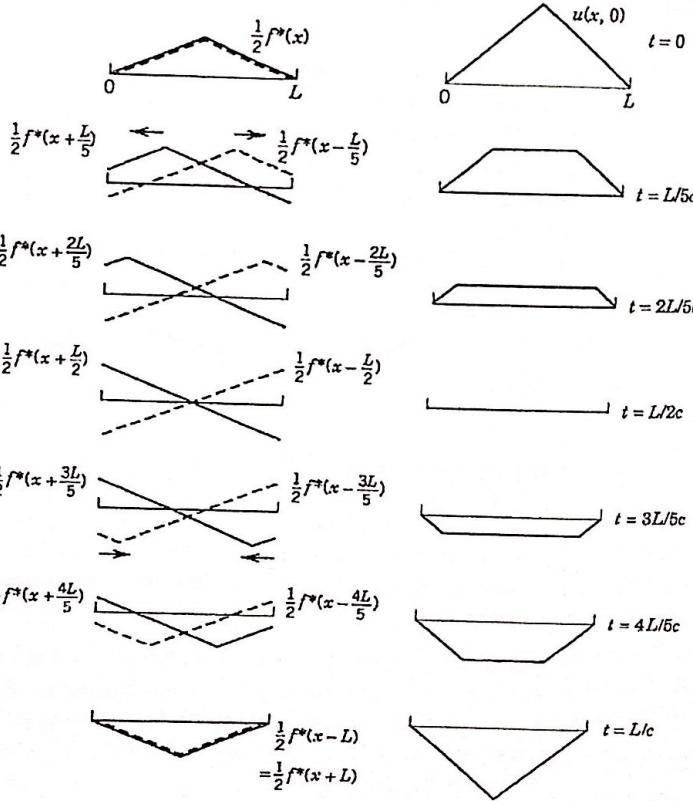


Fig. 288. Solution $u(x, t)$ in Example 1 for various values of t (right part of the figure) obtained as the superposition of a wave traveling to the right (dashed) and a wave traveling to the left (left part of the figure)

$$\frac{\partial u}{\partial t} = C^2 \nabla^2 u = C^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$



معاملات حرارت

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

معاملات حرارت يك بعد

$$u(0, t) = u(L, t) = 0$$

شرط ابز

$$u(x, 0) = f(x)$$

شرط اولی

بنابراین

$$u = X(x)T(t) \Rightarrow u_{xx} = X''(x)T(t) \quad u_t = X(x)T'(t)$$

$$\Rightarrow X(x)T'(t) = C^2 X''(x)T(t) \Rightarrow \frac{X''(x)}{X(x)} = \frac{T'(t)}{C^2 T(t)} = \lambda$$

و تران دهنده معتبر: جواب ثابت صفری ندارد. بنابراین $\lambda \geq 0$ تران ففرکر

$$\Rightarrow X(x) = a \cos kx + b \sin kx \stackrel{x=0}{\Rightarrow} a = 0$$

$$\stackrel{x=L}{\Rightarrow} b \sin kL = 0 \Rightarrow k = \frac{n\pi}{L}, n \geq 1$$

$$\Rightarrow T'(t) = -C^r \left(\frac{n\pi}{L}\right)^r T(t)$$

$$\Rightarrow T(t) = \alpha_n e^{-\lambda_n t}$$

$$\Rightarrow u_n = \alpha_n e^{-\lambda_n t} \sin\left(\frac{n\pi}{L} x\right)$$

$$\Rightarrow u = \sum_{n=1}^{\infty} \alpha_n e^{-\lambda_n t} \sin\left(\frac{n\pi}{L} x\right)$$

$$u \text{ ایسا سرطان } \Rightarrow f(x) = u(x, 0) = \sum_{n=1}^{\infty} \alpha_n x^n \sin\left(\frac{n\pi}{L} x\right)$$

$$\Rightarrow \alpha_n = \frac{1}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

$$u(x, t) = \sum_{n=1}^{\infty} \alpha_n e^{-\lambda_n t} \sin\left(\frac{n\pi}{L} x\right)$$

جواب محدله حرارت
با شرط اولیه را (0 میله)